Wall Effects on the Sedimentation Velocity of Suspensions in Viscous Flow

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The sedimentation velocity of multiparticle suspensions was investigated experimentally for cases where the column wall is likely to have a nonnegligible effect. The experiments demonstrate that the container wall diameter influences the falling velocity only for dilute systems, which is in contrast with a widely used, established empirical relationship.

Introduction

The settling of particles in a fluid generally takes place in a container of finite dimensions that could be expected to have some effect on the sedimenting velocity. This effect is bound to be related to the size of the solids relative to the container, d/D, and it is likely to be more relevant for dilute systems, where particle–particle and particle–wall hydrodynamic interaction effects could be of the same order.

We will consider exclusively viscous flow conditions, therefore, analysis and experiments will not be affected by changes in the Reynolds number. For the viscous flow regime the hindering influence of the container wall on a single particle falling in a tube has been analyzed theoretically, whereas the extension to suspensions of particles still relies on experimental evidence.

Single-particle suspension

This system has been studied quite extensively over the past hundred years and many correlations, both fully theoretical and empirical, have been recommended. The empirical equation of Francis (1933)

$$\frac{u_t}{u_{t\infty}} = \left(\frac{1 - d/D}{1 - 0.475 d/D} \right)^4 \tag{1}$$

provides a very good synthesis of the experimental data presented in the literature.

Dilute suspensions

No experimental studies appear to have been carried out explicitly to estimate wall effects on the settling velocity of dilute suspensions. In contrast, many publications report sedimentation velocities obtained in vessels of diameter much greater than the particle diameter, generally at least two or three orders of magnitude greater. These measurements for low particle concentration systems, pose practical difficulties, particularly with regard to the diffuse solid–fluid interface that arises from small variations in particle size and from hydrodynamic diffusion (Davis and Hassen, 1988). Such difficulties may explain the inconsistencies in the experimental data collected by Famularo and Happel (1965).

Recently the use of more advanced experimental techniques, involving, for example, light transmission, have enabled more reliable measurements to be carried out (Davis and Birdsell, 1988; Ham and Homsy, 1988; Al-Naafa and Selim, 1992). These experimental observations can be represented quite well by the theoretical equation (Batchelor, 1972)

$$\frac{u}{u_{t\infty}} = 1 - n(1 - \epsilon). \tag{2}$$

The theoretically predicted value for the coefficient n is 6.55 for perfectly monosized mixtures and somewhere around 5.5 for slightly dispersed mixtures possessing a large Peclet number (Batchelor and Wen, 1982), as is the particular system studied in this work.

Concentrated suspensions

The sedimentation velocity of concentrated multiparticle suspensions is well described by the empirical Richardson and

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Zaki (1954) equation

$$\frac{u}{u_{t\infty}} = k\epsilon^n, \quad (3)$$

where k and n are empirical parameters; in the original Richardson and Zaki (1954) work, both k and n were found to be functions of the flow regime and, for some specific case, of the ratio d/D. Equation 3 gives the same dependency of u on ϵ for values of the suspension voidage approaching unity as the theoretical Batchelor equation (Eq. 2).

The Richardson and Zaki (1954) equation, now forty years old, has enjoyed an unrivaled success over other correlations. The reason for this popularity is to be found in its extreme simplicity coupled with a generally satisfactory agreement with experimental observations.

Although wall effects were not a primary objective of the Richardson and Zaki investigation, they report correlations of n with d/D which, for the sedimentation in viscous flow, takes the simple form:

$$n = 4.65 + 19.5 d/D. \quad (4)$$

For this fluid dynamic case the parameter k was found always to equal 1 regardless of the value of the wall effect ratio, d/D.

These results are widely accepted and used. However, if we now consider the sedimentation velocities of two otherwise identical solid systems at a low Reynolds number that differ only with regard to the parameter d/D, we found that the Richardson and Zaki equation predicts the behavior depicted in Figure 1. This figure reveals an unexpected characteristic: Wall effects appear more important for high particle

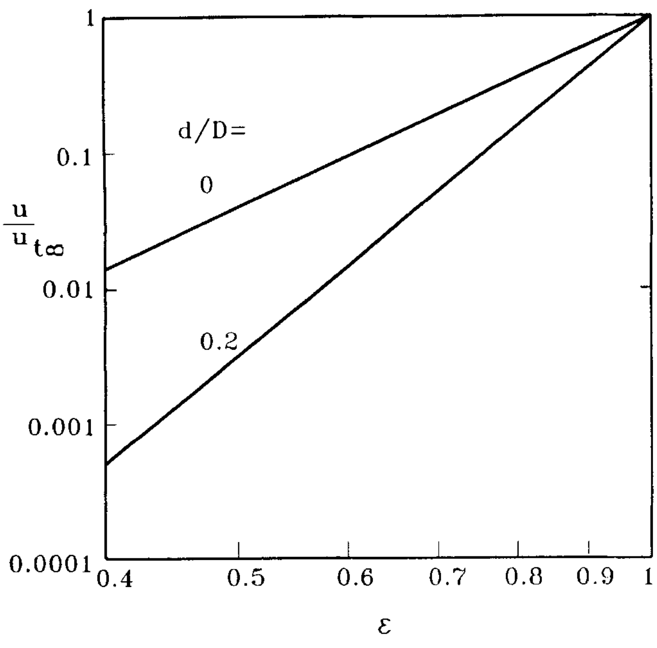

Figure 1. Sedimentation velocities in viscous flow regime as predicted by the Richardson-Zaki equation when wall effects are taken into account.

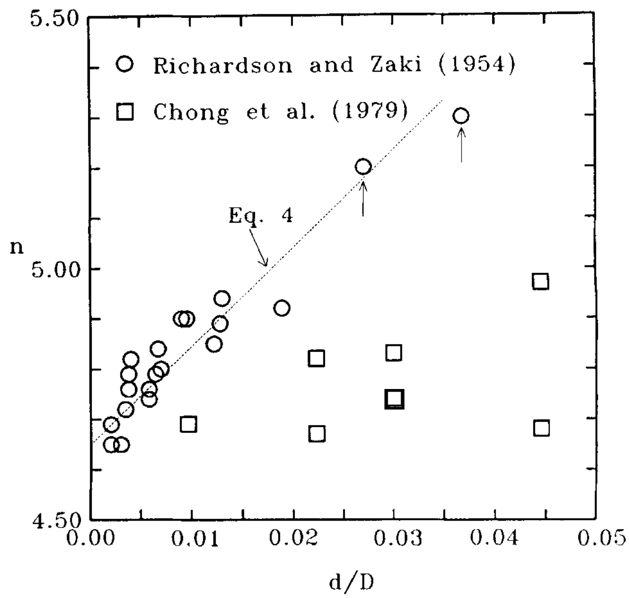

Figure 2. Reported values of the parameter *n* function of *d/D*.

volume concentration systems than for dilute systems and completely absent for the limiting condition of single-particle settling.

These strong wall effects at a high particle concentration do not appear to have been confirmed by successive work, as, for example that of Chong et al. (1979). Figure 2 depicts the experimental values of n as a function of d/D from these two studies. This depiction reveals an interesting feature: If the two experimental values found by Richardson and Zaki at the highest d/D are ignored (in the figure they are indicated by an arrow), then one could reasonably conclude that the wall hardly effects n at all, its value remaining roughly constant somewhere between 4.7 and 4.9.

Finally, it should be reported that, like Richardson and Zaki (1954), Chong et al. (1979) found k to be independent of d/D, although its value was close to 0.9 rather than 1.

Experimental

For this study 4.9-mm acetate spheres of a density of 1,280 kg/m^3 were utilized. The particle diameter was determined by averaging 50-μm measurements. Here the difference between one measured diameter and the average was always smaller than 0.05 mm. The fluid was a mixture of water and glycerol, and all measurements were conducted in a thermostatic bath at a temperature of 20°C. At this temperature the fluid density was 1,228 kg/m^3 and the fluid viscosity 0.155 kg/m/s (both quantities experimentally determined).

Four transparent cylindrical columns were utilized, all 500 mm tall, of internal diameter of 24, 40, 74 and 107 mm; d/D ranged from 0.046 to 0.205, the upper values being much higher than those previously investigated.

With the obvious exception of the single-particle case, where a sphere was dropped at the axis of the tube, each experiment was carried out by thoroughly mixing a known amount of fluid and particles in the sedimenting tube by rotating the tube axially and laterally until a satisfactory homo-

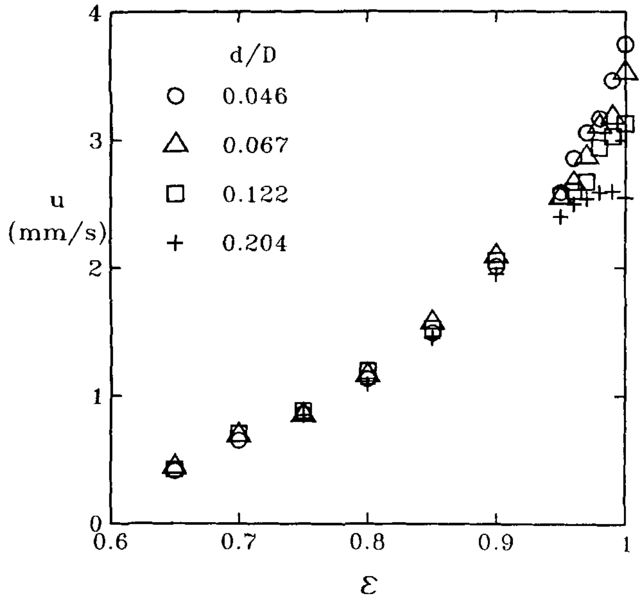

Figure 3. Experimental suspension-settling velocities.

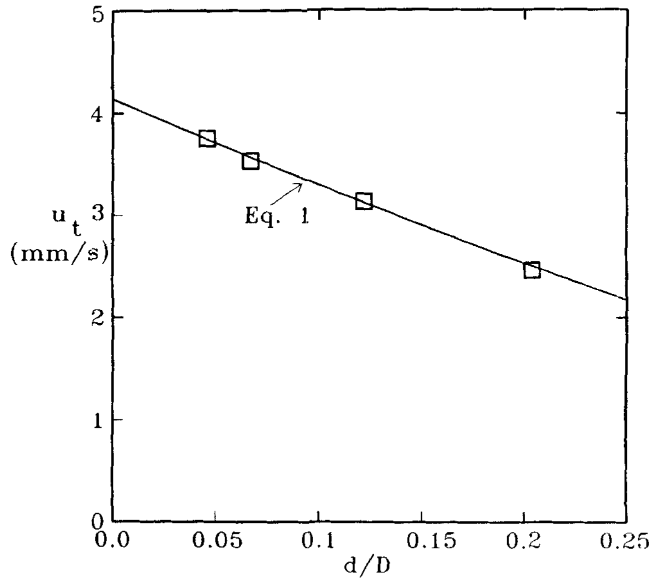

Figure 4. Experimental values of u_t compared with Francis empirical correlation.

geneity was achieved. The falling velocity of the sharp-suspension–clear-fluid interface was then measured by means of a stopwatch and the graduation lines on the column. The interface was always sharp and easy to pinpoint, the only exception being the more dilute suspension (at a voidage of 0.99): in that case, individual particle, as well as interface, settling velocities were measured.

Measurements for one system condition were repeated at least five times and the average value utilized in subsequent calculations. Deviation from the average was never more than 5%; as in the recent work of Nicolai and Guazzelli (1995), *d/D* did not seem to affect the magnitude of the deviation.

Results and Discussion

Figure 3 summarizes the results of all the experimental runs. It is evident that the container influences the falling velocities of the spheres in an appreciable manner only for dilute conditions, that is to say, for suspension voidages of over 0.95.

Single-particle suspensions

Measured falling velocities of a single sphere are shown in Figure 4 as a function of *d/D*. Equation 1 provides an excellent fit; u_{t_∞}, obtained by extrapolating the experimental velocities, is found to equal 4.15 mm/s, in satisfactory agreement with the theoretical Stokes' law value:

$$u_{t\infty} = \frac{(\rho_p - \rho)d^2 g}{18\mu} = 4.35 \text{ mm/s}. \tag{5}$$

Equation 1 was therefore utilized for the determination of u_t in the discussion that follows.

Dilute suspensions

In Figure 5 the observed sedimentation velocities for $\epsilon > 0.95$ are depicted on a larger scale, which shows the effect of the container diameter to be considerable.

It has been assumed that Richardson and Zaki's equation, Eq. 3, adequately represents the experimental data; the values of n and k were obtained by linear regression. These values are plotted in Figures 6 and 7, respectively.

Figure 6 shows a decrease of the value of n with increasing *d/D*, a trend opposite to that reported by Richardson and Zaki (1954), Eq. 4. For the lowest value of *d/D*, that is, for cases where the wall has the minimum influence, the experi-

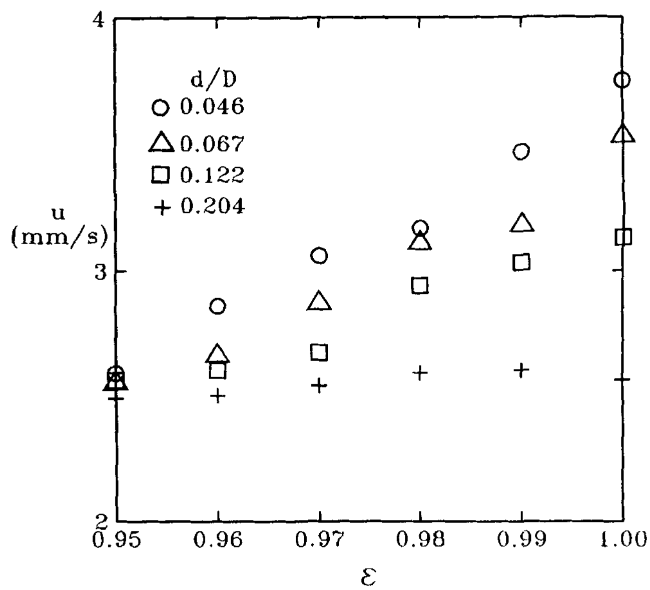

Figure 5. Experimental sedimentation velocities for dilute systems.

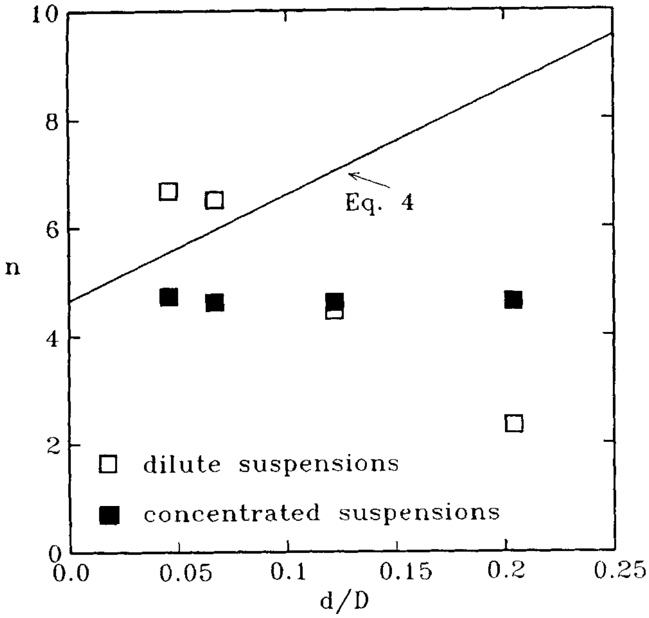

Figure 6. Experimental values of n for dilute and concentrated systems compared with the Richardson–Zaki correlation.

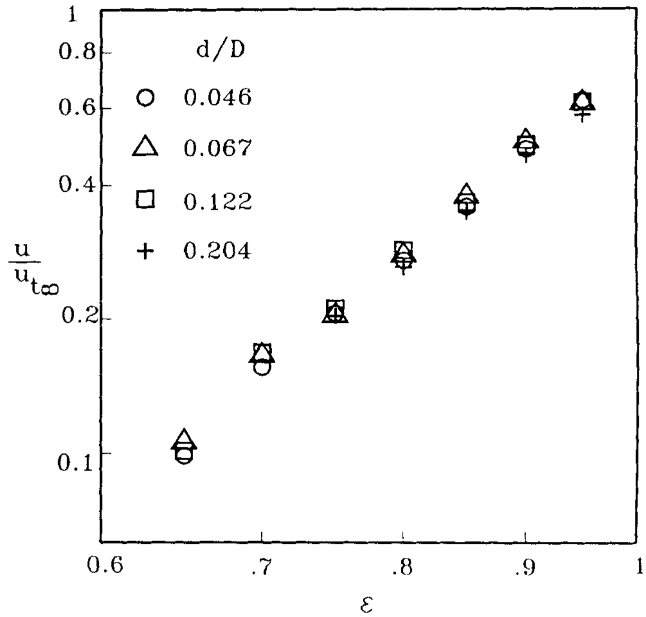

Figure 8. Experimental sedimentation velocities for concentrated systems.

mental n value was found to be close to 6.5. If we take the uncertainties connected with these measurements into account, the comparison with the theoretical value of 5.5 is quite satisfactory.

Because Figure 7 indicates a good correspondence between k and the values of u_t/u_{t_∞}, these results can be summarized by saying that, for dilute suspensions, the sedimentation velocity of spheres in the creeping flow regime is given by the following relationship:

$$\frac{u}{u_t} = \epsilon^n, \tag{6}$$

with n decreasing sharply as d/D increases.

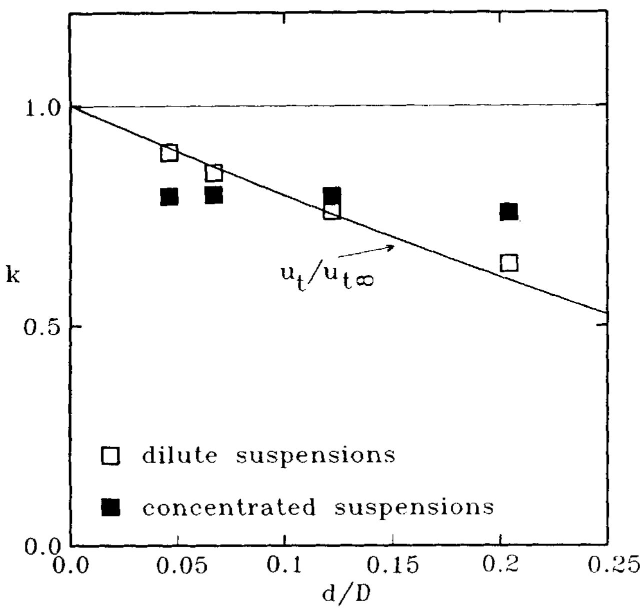

Figure 7. Experimental values of k for dilute and concentrated systems compared with u_t/u_{t_∞}.

Concentrated suspensions

Figure 8 shows the conventional representation of the settling velocity–voidage results for the concentrated suspension on logarithmic coordinates. For all four cases it can be seen that this results in very good straight lines, thereby confirming that Eq. 3 is a good representation of the observed behavior. The least squares minimization routine again yielded the best values of n and k; these values are also plotted in Figures 6 and 7.

From these two figures it is evident that, for concentrated suspensions within the range of investigation, the container wall has no effect on the sedimentation velocity, as both n and k appear quite independent of d/D.

Figure 6 shows that the effect of d/D on n suggested by Richardson and Zaki is quite unreliable and its extrapolation to high values of d/D extremely dangerous. The results of the present study (in which n is found to range from 4.60 to 4.75) are more in line with the evidence presented by Chong et al. (1989)), in which the value of n was found to vary between 4.67 and 4.97 for a narrower range of particle-to-wall-diameter ratio than was considered in the present study (d/D was never larger than 0.045, see Figure 2).

Figure 7 reveals that k cannot be correlated with u_t over the range investigated (u_t varying by a factor of 1.5). On the other hand, at the present stage no explanation can be given as to why k is always 0.8, which is in contrast with the experimental work of Richardson and Zaki (1954) (where k was found to be practically 1), but closest to the results of Chong et al. (1989), who found k to be roughly 0.9.

Further investigation of this issue is clearly needed. It is interesting to note, however, that Richardson and Zaki (1954) utilized solids that, with only one exception, ranged in size from 0.1 to 0.35 mm, whereas in Chong et al. (1989), the particles were 0.5 to 1.1 mm in diameter, and in this study the sphere diameters were 4.9 mm. We could speculate that particle size has some influence on k; however, to our knowledge, no theoretical analysis exists to support this tentative explanation.

Conclusions

Equation 3 (universally known as the Richardson and Zaki equation) is a very good representation of the sedimentation velocities of uniform solids in the viscous flow regime, both for dilute and concentrated suspensions. However, the correlations originally recommended, and which are widely used, for the parameters n and k were found to be quite inappropriate as far as the effect of the container wall is concerned.

For concentrated systems (voidages up to around 0.95), both n and k are independent of d/D: n was found to be in line with previous studies, in the range of 4.6 to 4.8, while k was found to be constant at a value of about 0.8. This latter result needs further investigation.

For dilute systems, both n and k were strong functions of d/D, n decreasing as the wall effect became more important, and k being well represented by the ratio between the relevant bounded single-particle settling velocity, u_t, and the unbounded single-particle settling velocity, $u_{t\infty}$.

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Notation

d = particle diameter, mm
D = tube diameter, mm
u_t = bounded single-particle settling velocity, mm/s
$u_{t\infty}$ = unbounded single-particle settling velocity, mm/s

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